



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

Let $y=mx+\sqrt{a^2m^2+b^2}$, $y=m'x+\sqrt{a^2m'^2+b^2}$ be the equation of the tangents parallel to two conjugate diameters. But between m and m' there is the well known relation: $mm'=-b^2/a^2$, whence $m'=-b^2/a^2m$. Substituting this in the second of the above equations, we have $a^2ym+b^2x=ab\sqrt{a^2m^2+b^2}$, and dividing the last equation by $y-mx=ab\sqrt{a^2m^2+b^2}$, we obtain

$$m=\frac{b}{a}\cdot\frac{ay-bx}{ax+bx};$$

and substituting in the latter equation, we finally get $a^2y^2+b^2x^2=2a^2b^2$, the equation of a concentric ellipse with the semi-axes $a/\sqrt{2}$ and $b/\sqrt{2}$.

391. Proposed by W. J. GREENSTREET, M. A., Editor, The Mathematical Gazette, Burghfield, England.

An ellipse is inscribed in the triangle of reference and has one focus at ($\sec A$, $\sec B$, $\sec C$). Find the other focus and the sum of the squares of the axes of the ellipse.

Solution by WILLIAM HOOVER, Ph. D., Athens, Ohio.

In trilinear coördinates, let (a_1, β_1, γ_1) , (a_2, β_2, γ_2) be the two foci. Then the first is found by

$$a_1\cos A=\beta_1\cos B=\gamma_1\cos C\dots(1),$$

$$\text{and } a a_1+b \beta_1+c \gamma_1=2 \Delta \dots(2),$$

$$\text{or, } a_1=2R\cos B\cos C, \beta_1=2R\cos A\cos C, \text{ and } \gamma_1=2R\cos A\cos B\dots(3),$$

R being the radius of the circum-circle.

If b_1 be the semi-minor axis of the ellipse,

$$a_1a_2=b_1^2=\beta_1\beta_2=\gamma_1\gamma_2\dots(4), \text{ or,}$$

$$\cos B\cos C.a_2=\cos A\cos C.\beta_2=\cos A\cos B.\gamma_2\dots(5),$$

which with (3) gives

$$a_2=R\cos A, \beta_2=R\cos B, \gamma_2=R\cos C\dots(6).$$

$$(4) \text{ now gives } b_1^2=2R^2\cos A\cos B\cos C\dots(7).$$

Also, d =the distance between the ortho-center and circum-center, is given by

$$d^2=R^2(1-8\cos A\cos B\cos C)\dots(8).$$

If $2a_1$ be the major axis of the ellipse, $a_1^2 = \frac{1}{4}d^2 + b_1^2 = \frac{1}{4}R^2$, or $2a_1 = R$, and determining $4a_1^2 + 4b_1^2$.

392. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.

A tangent to a curve at any point P cuts the tangent and the normal at a fixed point O in the points M and N , and the rectangle $OMP'N$ is completed. Find the curve which is such that the triangle formed by the tangents at any three points P, Q, R is equal to the triangle formed by the corresponding points P', Q', R' .

No solution of this problem has been received.

CALCULUS.

316. Proposed by C. N. SCHMALL, New York City.

$$\int_0^\infty \frac{\cos ax}{1+x^2} dx = \frac{1}{2} \pi e^{-a} = \int_0^\infty \frac{x \sin ax}{1+x^2} dx.$$

(From Bromwich, *Theory of Infinite Series*, p. 442, ex. 5, and also from Carslaw, *Fourier's Series*, p. 113, ex. 12.) Prove this by any method.

II. Solution by W. J. GREENSTREET, M. A., Editor of the Mathematical Gazette, Burghfield, England.

Lemma. $u = L_{h \neq 0} [\sin h + \frac{1}{2} \sin 2h + \frac{1}{3} \sin 3h + \dots] = L_{h \neq 0} \frac{\pi - h}{2}$, the sum being taken between 2π and small values of h , $= \frac{1}{2} \pi$.

$$\therefore u = \int_0^\infty \frac{\sin x}{x} dx = \frac{1}{2} \pi, \text{ and it is also clear that } \int_0^\infty \frac{\sin ax}{x} dx = \frac{1}{2} \pi.$$

$$\text{Now let } U = \int_0^\infty \frac{x \sin ax}{1+x^2} dx; \quad U - \frac{1}{2} \pi = \int_0^\infty \frac{x \sin ax}{1+x^2} dx - \int_0^\infty \frac{\sin ax}{x} dx$$

$$= - \int_0^\infty \frac{\sin ax}{x(1+x^2)} dx \dots (1).$$

Differentiating twice with respect to a ,

$$\frac{d^2 U}{da^2} = \int_0^\infty \frac{x \sin ax}{1+x^2} dx = U.$$

$$\text{Multiplying by } \frac{dU}{da}, \quad \frac{d^2 U}{da^2} \cdot \frac{dU}{da} = \frac{U \cdot dU}{da} \cdot \frac{1}{2} \frac{d}{da} \left(\frac{dU^2}{da^2} \right) = \frac{1}{2} \frac{d}{da} (U^2), \quad \left(\frac{dU}{da} \right)^2 = U^2 + \kappa,$$

$$\frac{dU}{\sqrt{U^2 + \kappa}} = da.$$